Research statement

Simon Henry

The following text is a brief summary of both my past research and my plans for future research.

My PhD thesis ([11]) was about topos theory and its relations to non-commutative geometry. It leads me to also think about questions of constructive mathematics, in order to use the internal logic of toposes, and about point-free topology, in order to be able to use tools of analysis in this framework of internal logic.

After my PhD, I gradually moved my interests to algebraic topology and higher category theory, but I still have some ongoing work and projects related to topos theory, point-free topology and non-commutative geometry. For this reason the following text will be divided in three independent parts each discussing a different area. The first one about higher category theory, and is my most active area of research at the moment.

1 Algebraic topology : Algebraic models for higher category theory

My interest in topos theory and categorical logic, and the year I spent as a post-doc in Nijmegen algebraic topology group naturally lead me to follow the development of the higher categorical version of topos theory and categorical logic, which consists mostly of the "homotopy type theory" program ([29]).

But at the moment we still do not have a satisfying ∞-categorical version of categorical logic :

The first problem is that we do not know how to formalize large parts of classical mathematics in homotopy type theory. One of the major obstruction here is that we do not know how to formalize higher category theory itself in homotopy type theory and this leaves us stuck as soon as there is a problem of higher coherence conditions to handle (and those problems are very common in homotopy type theory). The classical approaches to higher category theory or higher coherence conditions is trough simplicial sets (or other similar combinatorics, cubical sets, opetopic sets, dendroidal sets, associahedrons,...), but in type theory, just defining what simplicial types are already lead to an apparently unsolvable higher coherence problem.

A possible solution to this has been suggested by G.Brunerie (see [6] or the appendix A of his PhD thesis [7]) : he showed that Grothendieck's definition of globular ∞-groupoids can be formalized within homotopy type theory. Unfortunately, even in classical mathematics we do not really know how to work with this definition : the comparison with topological spaces or other models for ∞-groupoids is still an open problem (known as the homotopy hypothesis), we do not know how to put a model category structure on the category of Grothendieck's ∞-groupoids, and we do not even know how to define the ∞-groupoids of functors between two ∞-groupoids, even in simple cases like constructing path spaces.

The second difficulty is that it is not clear either that intentional type theory is indeed the internal logic of higher categories. One of the problem is that the higher categorical structures that arises from type theory are very naturally globular (see for example [28]). It would be more accurate to say that intentional type theory is the internal logic of globular ∞-categories, but as I mentioned above, comparing globular higher categories to other models for higher categories is still an open problem. The other problem is that the most natural notion of "model of type theory" (contextual categories with additional structures) are structures that are a lot more "strict" than most structures considered to model ∞-categories, so we need some sort of strictification theorem to compare them.

These remarks lead me to work first on the theory of globular models for higher category, with the goals of both being to work with them and to prove that they are equivalent to other known models.

My results on this subject are in [16], where i focused on the special case of ∞-groupoids (instead of $(\infty, 1)$-categories). I studied what I called "cylinder categories" which are just the categorical opposites of the path categories introduced by I.Moerdijk and B.Van Den Berg in [5], which provides a categorical semantic for a very weak fragement type theory. More precisely I have been interested in a special kind of cylinder categories that are freely generated in a certain sense and that I have called "cylinder coherators" because they play a role similar to Grothendieck's coherator in the definition of ∞-groupoids. My results are :

- Each cylinder coherator $\mathcal{C}$ defines a notion of weak ∞-groupoids called the $\mathcal{C}$-groupoids. This is a completely algebraic notion.
- For each $\mathcal{C}$, the category of $\mathcal{C}$-groupoids is a combinatorial semi-model category in which every object is fibrant. These semi-model categories are all naturally Quillen equivalent to each other.
- For certain $\mathcal{C}$ one gets "simplicially shaped" groupoids (for example semi-simplicial algebraic Kan complexes). These can be easily compared to simplicial sets or topological spaces.
- There is a cylinder coherator $\mathcal{C}$ such that $\mathcal{C}$-groupoids are globular sets endowed with all the operations that one can define in intentional type theory using only (weak) identities type. This provide the first globular definition of ∞-groupoids for which one can prove the homotopy hypothesis. Also, G.Brunerie's argument can also be applied to this kind of groupoids to show that they can be defined within homotopy type theory.
- Under a seemingly simple technical conjecture, there is a cylinder coherator $\mathcal{C}$ such that the $\mathcal{C}$-groupoids are exactly Grothendieck's ∞-groupoids. Hence, if this conjecture holds, this would prove the homotopy hypothesis, as well as solve most of the open problems related to Grothendieck's definition of groupoids (for example, constructing an ∞-groupoid of weak functors between two ∞-groupoids).

This seems to be a very promising direction in order to solve the problems mentioned above. In particular, this new definition of ∞-groupoids based on type theory has all the good properties that we want : it satisfies the homotopy hypothesis, has all the good properties expected for a category of ∞-groupoids, every objects in every model of intentional type theory carries the structure of a groupoid in this sense, and finally it can be defined within homotopy type theory. A definition of ∞-categories with the same sort of properties would be a perfect candidate to solve all the problems mentioned above. This paper is only the start of a larger research program, here is a first list of things that should be investigated and on which I have been working recently :

- One needs a similar notion of "coherator" for a definition of $(\infty,1)$-category and for other sorts of higher categorical structures (∞-categories, monoidal $(\infty,1)$-categories, A_∞ and E_n algebras, ∞-operads, etc.), which would allow to similarly define globular notion of $(\infty,1)$-category and to compare them to usual notions. Good starting points for this are the axiomatics approach to (∞,n)-category by C.Barwick and C.Schommer-Pries in [4] or the synthetic approach to $(\infty,1)$-category by M.Shulman et E.Riehl in [26].
- The theory of $\mathcal{C}$-groupoids for a coherator $\mathcal{C}$ seems to be mostly constructive (which is important if we want to have it works within type theory), but this has not been checked in details as our paper [16] was mostly focused on trying to prove a comparison theorem with classical (and non constructive) models for homotopy types. There is at least one difficulty to be addressed here. Because of the existence of non hyper-complete ∞-toposes, it is expected that in constructive mathematics, there is three non equivalent notions of equivalences between ∞-groupoids : being a bijection on all the π_n, being invertible as an anafunctor (the distinction between the first two does not appears for n-groupoids for $n < \infty$) and being invertible as a weak functor. As this feature only appears in constructive mathematics, it has not been treated yet.
- One needs to show that those definitions of ∞-groupoids can indeed be used within type theory. Although one can convince oneself that it works, it is very likely that only a computer implementation would be a convincing proof of this. Next one need to see how much of standard algebraic topology can be internalized in type theory using this notion of ∞-groupoids.
- And of course one can try to prove Grothendieck's homotopy hypothesis, there is several possible approaches to this question, it is not clear which ones is the more promising.

At an abstract level, the inner working of this theory of coherators relies on two things : First a kind of "weak model structure" on a category of "pre-cylinder categories" where the cylinder categories mentionned above are the fibrant objects and the cylinder coherator are the fibrant replacement of the free pre-cylinder category on one object. And second a process that complete any cylinder category into a (weak) model category, and such that the weak equivalence in the sense of the weak model structure on pre-cylinder category induces Quillen equivalence between the completion.

This weak model structure can be thought of as a homotopy theory of "higher cartesian theories" and the completion into a model category as looking at the "category of models" of the theory, and to some extent the general goal of this work is to build a framework to help understand most of the construction of model structures used in algebraic topology and higher category theory and of the various comparison theorem between those models in a more systematic way (almost all the "algebraic" model structure used in algebraic topology, like those on simplicial sets, cubical sets, dendroidal sets, simplicial spectra and so one appears in this framework).

At a present time the theory is efficient at studying and comparing "fully weak higher theory" where there is no equations between the various operations of the theory, but only other operations that produces "isomorphisms" or "homotopies" in some sense (Because these corresponds to the fibrant-cofibrant objects of our weak model structure). Most of the "higher theory" that peoples use in practice (everything based on simplicial, cubical or dendroidal sets for example) are not fully weak (for example for simplicial sets the relation between the degeneracies are relations between operations) only Globular models, models based on semi-simplicial sets or semi-dendroidal sets as well as some opetopic models are fully weak.

What I'm interested in at the present time is the understanding of how equivalences between such fully weak models and "semi-strict" models are obtained, i.e. some general "strictification results". There is a lot of examples of such strictification results which are proved, and even some very general ones : Mac Lane coherence theorem for monoidal categories and the various results of strictification for pseudo-algebras for 2-monads, the Berger-Moerdijk strictification theorem for homotopy algebras of Σ-cofibrant operads, the equivalence between quasi-categories and simplicial categories, the equivalence between semi-simplicial sets and simplicial sets for representing homotopy types, the comparison by K.Szumilo between Brown category of fibrant objects and finitely complete quasicategory etc... and on the other hand, a lot of similar question that are open problem : Proving that various form of intentional type theory are the internal logic of different type of Higher categories (known in some cases), the various form of C.Simpson semi-strictification conjecture, a very general semi-strictification conjecture of M.Batanin for globular operads whose slices are Σ-cofibrant, etc. It appears that the ideas on all these examples are always very similar and there is real hope to obtain general strictification theorems. Moreover there is also a long list of "counterexamples" and cases where strictification is impossible, which can help us understand more precisely what are the hypothesis needed.

Recently I have been working more concretely on C.Simpson's semi-strictification conjecture to test those ideas. What is interesting for this conjecture is that we have a famous erounous paper [24] by M.Kapranov and V.Voevodsky that is supposed to contain a sketches of proof of this conjecture. In my recent preprint [21] I have investigated in detail this paper of Kapranov and Voevodsky and I explain what is the main obstruction for their strategy to work and I Proved new results about the theory of polygraph that allow to overcome this obstruction. I'm curently working to extend this into a full proof of C.Simpson conjecture.

2 Toposes, C^*-algebras and K-theory

This has been my main research interest during my PhD thesis. The starting idea was that both topos theory and non-commutative geometry are concerned with certain "generalized spaces" and in a lot of situations some of these "generalized spaces" have been (or can be) studied both by attaching to them a topos or a C^*-algebra (e.g. foliations, dynamical systems, groupoids, graphs, etc.). My goal is to obtain a better understanding of the relation between the two theories : how does the information extracted from the topos and the algebra relates ? Can the algebra be constructed from the topos, or conversely ? Can we transport some techniques and ideas from one subject to the other ?

My first approach to this question was to use a good notion of continuous field of Hilbert spaces on a topos (or more generally of Banach spaces or of C^*-algebras) using the internal logic : it is just an ordinary Hilbert space in the internal logic of the topos. These objects have already been studied in the 80s (by B.Banachewski, C.J.Mulvey, C.W.Burden and others) and it was known that over ordinary topological spaces they are equivalent to the usual definitions of continuous fields in functional analysis. Moreover these continuous fields can be handled easily using internal logic : any constructive result about Hilbert spaces translates automatically into a result about continuous fields.

The important point is that the algebra of endomorphisms of such a continuous field of Hilbert spaces is a C^*-algebra which is naturally attached to the topos. It appears that those algebras are not exactly those corresponding to the analogy above if one starts with an object which can be represented by both a topos and an algebra, but it is close enough to read some properties of the correct algebra on the topos and conversely. My thesis [11] and some of my following work was devoted to this approach.

The most satisfying result that I obtain in this direction is a construction in [19] of a $*$-algebra naturally attached to any "locally absolutely compact" topos. This $*$-algebra can be completed into either a reduced or a maximal C^*-algebra as well as a Banach L^1-algebra, this construction applies to all the examples that we have of objects to which both a C^*-algebra and a topos can be attached and produces the correct C^*-algebras, at least up to Morita equivalence. Moreover the algebra comes with a universal property : Modules over the

$*$-algebra are the same as sheaves of module over the sheaf of continuous functions (with values in $\mathbb{R}$ or $\mathbb{C}$) on the topos.

But before that I obtained several other results of independent interest :

- In the first chapter of my thesis I have studied this construction in the case of atomic toposes where everything can be understood completely explicitly in terms of a certain notion of "hypergroupoid". This case served as a "toy model" for large part of my subsequent work.
- In [18], I compared topos theory and non-commutative geometry at the level of "measure theory". In non-commutative geometry, measure theory corresponds to the theory of Von Neumann algebras, in topos theory, it is played by a new notion of integrable boolean toposes and the main result of the paper is a construction of an analogue of the "modular time evolution" (a typical feature of von Neumann algebras) for integrable boolean locally separated toposes. It takes the form of a completely canonical $\mathbb{R}^{>0}$-fiber bundle on those toposes, that generates a one parameter family of endofunctors of the category of fields of Hilbert spaces over the topos. If the bundle is trivial, then the family of endofunctors is also trivial, but in this case any section of that bundle provides an "invariant measure" on the topos (the analogue of a trace in non-commutative geometry).
- [15] is a sketch of a technique to reconstruct a topos from its category of continuous fields of Hilbert spaces endowed with its monoidal structure. It completely proves the result for boolean (locally separated) toposes, but the result of [14] mentioned below should allows one to extend this to a considerably larger class of (non-boolean) toposes.
- In [14] I proved a technical result in topos theory : that under a purely topological assumption on a topos (essentially separation) one can construct objects in the topos that are internally "finite". Those finite objects are very important when doing analysis over the topos, in fact the version of this result for boolean toposes (considerably easier) is a key lemma in [18] and [15] and this result is also crucial for the next two papers mentioned.
- In [12] I proved that for a topos that is decidable separated and locally compact, one has a "Green-Julg type" theorem asserting that its category of continuous fields of Hilbert spaces is equivalent to the category of Hilbert modules over a "C^*-algebra of the topos". In the case where the theorem applies, it indeed constructs the correct C^*-algebra to be attached to the topos in the sense that it fits with all the examples of objects producing both a topos and a C^*-algebra. Unfortunately, the cases where a theorem of this kind holds are essentially the "not very interesting" ones : they basically correspond to proper Hausdorff groupoids, and will for example only produces type I (or post-liminal) algebras. This result was important toward the more general construction of the $*$-algebra of a topos obtained in [19] that I mentioned above. A large part of the paper is devoted to the definition and study of a notion of categorical (co)completeness adapted to C^*-categories, and it also contains a constructive treatment of parts of the theory of C^*-algebras and C^*-categories that hasn't been developed before.

The next step in this direction, now that we know how to attach a C^*-algebra to a reasonable topos (in fact both a reduced and a maximal C^*-algebra), is to investigate how the properties of the topos and the algebra relates. Here we have mostly in mind (co)homological and K-theoretic properties.

For example the Baum-Connes conjecture corresponds (at least on examples) to a comparison between the K-theory of the reduced algebra of a topos and a kind of K-theory attached to the topos itself. One of my goals is to give a better topos theoretic, and higher categorical formulation of this. The first step is to define KK-theory relative to a topos (for fields of C^*-algebras over a given topos), generalizing equivariant KK-theory. Next, using the topos theoretic Green-Julg theorem of [12], one should be able to construct a Kasparov descent homomorphism and, finally, one should be able to formulate a topos theoretic version of the Baum-Connes conjecture. One should also be able to do the same things for the algebraic convolution algebra and for the Banach L^1-algebra to get an analogue of the Farrell-Jones conjecture and of the Bost conjecture in this framework. We believe that this will be especially interesting for the Farrell-Jones conjecture which has never been formulated outside a purely algebraic framework.

I'm also very interested in reformulating these isomorphisms conjecture in a language closer to modern algebraic topology and higher category theory, a little bit in the spirit of [8].

Similarly, for the C^*-algebra of a foliation, it is known that the cohomology of the corresponding topos (the leafwise cohomology) is the same as the cyclic cohomology of the sub-algebra of smooth functions of the

C^*-algebra. It would be interesting to understand this result from the topos theoretic perspective and to see if it can be extended to more general toposes.

Another more elementary point, is that it also seems that the non-commutative measure theory of the C^*-algebra is related to the theory of cosheaves of Banach spaces on the topos. This is also something that we need to understand. If it is the case, it could then be used to study some sort of non-commutative measure theory for toposes that does not give rise to a C^*-algebra, for example for the toposes coming from algebraic geometry or those with some non-locally compact isotropy groups.

Finally, I have also been thinking about a construction of a C^*-algebra attached to a smooth stack.

A smooth stack is a kind of generalized manifold. To any Lie groupoid, one can associate a smooth stack which is essentially the space of orbits of the groupoid, and up to Morita equivalence the C^*-algebra of the groupoid only depends on this stack (and if one fixes the natural map from the space of objects of the groupoid to the stack then the algebra is determined up to unique isomorphism). But it appears that there is a lot of situations where one has a smooth stack which does not come from a groupoid but which admits a C^*-algebra. A typical example of this is the C^*-algebra of G.Skandalis and I.Androuliakis attached to a singular foliation (see [2]). Moreover it has been shown in [27] that one can attach a smooth stack to any Lie algebroids (integrable or not). It would be interesting to investigate the C^*-algebra of those stacks (in the case of an integrable Lie algebroids it gives the C^*-algebra of the corresponding Lie groupoid).

This construction relies on ideas inspired from the construction of the C^*-algebra of a topos. There are two strategies for the construction of the algebra attached to a stack, which are hopefully equivalent : the first is to represent the stack as a quotient stack of a groupoid whose space of morphisms is itself already a stack. The main point is then that under certain conditions one can define what is a density on a stack and one can hence try to apply the construction of the groupoid algebra based on half-density for this groupoid (this is exactly what is done in [2] although stacks are not mentioned explicitly there). The second strategy is to represent the stack as a colimit of a diagram of manifolds and submersions between them, turn that diagram into a a diagram of C^*-algebras and correspondences (using the "relative half-density" correspondence attached to a submersion) and investigate if the corresponding colimit exists in the category of C^*-algebras and correspondences (co-limit of this kind have been investigated in [1]). These two approaches seem to agree on all the examples where I have been able to try both of them.

C^*-algebras of Lie groupoids (and smooth stacks) are very important, for example the construction of the algebra of a Lie groupoid is one way to mathematically understand quantization (see [25]), and from this perspective, smooth stacks might appear as a more suitable framework to apply the ideas of [22] in physics than topos theory.

3 Point-free topology, locales theory

Locales are an alternative to topological spaces. While a topological space is given by a set of points and a set of "open subsets" satisfying some stability properties, a locale is defined by just specifying a set of "open subspaces" with some operations (arbitrary unions, finite intersections), and its set of points is recovered afterward, hence the name "point-free topology". There is a close relationship between topological spaces and locales (an adjunction which induces equivalences on very large sub-categories) but the theory of locales introduces some new interesting objects, which might have no points (like the space of generic real numbers) and is a little better behaved than the category of topological space in some respect. One can consult [23] for a non-technical overview of the subject.

Point-free topology has some interest in classical mathematics, but it is especially interesting regarding constructive mathematics and topos theory : the relation between toposes and topological spaces goes trough the notion of locales, and in constructive mathematics the notion of locales is considerably better behaved than topological spaces. Indeed a lot of classical spaces will "lack of points" without the axiom of choice (typically infinite products, functions spaces, etc.) and the point-free perspective handles them considerably better. A lot of theorems considered as completely non-constructive (like the Tychonov theorem, the Hahn-Banach theorem etc.) become constructive once formulated in the language of locales.

My main contribution to the subject was proving in [17] a localic and fully constructive version of the Gelfand duality, conjectured by C.J.Mulvey and B.Banaschewski in [3], which is an equivalence between compact Hausdorff locales and a new notion of localic commutative unital C^*-algebras. This just gives back the ordinary

Gelfand duality when interpreted in classical mathematics, but it can also be interpreted in other constructive models, like the category of sheaves over a space, where it gives a statement relating semi-continuous fields of commutative C^*-algebras over a given space X with proper and relatively separated covers of X, without any assumption on the base space X. In order to achieve this I developed a constructive theory of metric locales and of their metric completions.

I have also extended this to non-unital algebras (and hence locally compact regular locales) in [10]. In [13] I have applied this to produce a better behaved version of the Bohr topos construction (introduced in the topos theoretic approach to Quantum physics) which solves some of the problems of this theory.

More recently, in [20], I have applied the ideas of point-free topology to the theory of isotropy of toposes developed by J.Funk, P.Hofstra and B.Steinberg in [9]. This theory constructs for each Grothendieck topos a completely canonical group object in the topos, called the isotropy group which acts canonically on every objects of the topos making all morphisms equivariant. I have shown that their isotropy group is in fact the group of points of a more general localic isotropy group, which do not just act on every objects of the topos, but also on every locales or toposes defined over this base topos. Large part of their theory still work in this localic framework, but the localic group has considerably better functoriality and stability property, which allow to push the theory further : this solves certain difficulties, for example it explains the phenomenon of higher isotropy observed in [9], and gives a new canonical factorization of geometric morphisms into “connected atomic morphisms” followed by “essentially anisotropic morphisms” which was one of the open problem of the theory.

I do not have any precise research plan regarding this aspect of my work, even though there are some questions I am interested in : trying to develop a good constructive and localic integration theory, and the related problem of developing a constructive theory of Von Neumann algebras parallel to the theory of localic C^*-algebras I have already developed, or to understand better the various type of “surjections” between locales and image factorization.

Références

[1] Suliman Albandik and Ralf Meyer. Colimits in the correspondence bicategory. *arXiv preprint arXiv :1502.07771*, 2015.

[2] Iakovos Androulidakis and Georges Skandalis. The holonomy groupoid of a singular foliation. *Journal für die reine und angewandte Mathematik (Crelles Journal)*, 2009(626) :1–37, 2009.

[3] Bernhard Banaschewski and Christopher J Mulvey. A globalisation of the Gelfand duality theorem. *Annals of Pure and Applied Logic*, 137(1) :62–103, 2006.

[4] Clark Barwick and Christopher Schommer-Pries. On the unicity of the homotopy theory of higher categories. *arXiv preprint arXiv :1112.0040*, 2011.

[5] Benno van den Berg and Ieke Moerdijk. Exact completion of path categories and algebraic set theory. *arXiv preprint arXiv :1603.02456*, 2016.

[6] Guillaume Brunerie. Syntactic grothendieck weak ∞-groupoids. *Unpublished note, available at uf-ias-2012. wikispaces. com*, page 191, 2013.

[7] Guillaume Brunerie. *On the homotopy groups of spheres in homotopy type theory*. PhD thesis, Université de Nice Sophia Antipolis, 2016. arXiv :1606.05916.

[8] James F Davis and Wolfgang Lück. Spaces over a category and assembly maps in isomorphism conjectures in K-and L-theory. *K-theory*, 15(3) :201–252, 1998.

[9] Jonathon Funk, Pieter Hofstra, and Benjamin Steinberg. Isotropy and crossed toposes. *Theory and Applications of Categories*, 26(24) :660–709, 2012.

[10] Simon Henry. Constructive Gelfand duality for non-unital commutative C^*-algebras. *arXiv preprint arXiv :1412.2009*, 2014.

[11] Simon Henry. *Des topos à la géométrie non commutative par l'étude des espaces de Hilbert internes*. PhD thesis, Université Paris 7, 2014.

[12] Simon Henry. Complete C^*-categories and a topos theoretic Green-Julg theorem. *arXiv preprint arXiv :1512.03290*, 2015.

[13] Simon Henry. A geometric Bohr topos. *arXiv preprint arXiv :1502.01896*, 2015.

[14] Simon Henry. On toposes generated by cardinal finite objects. *arXiv preprint arXiv :1505.04987*, 2015.

[15] Simon Henry. Toward a non-commutative Gelfand duality : Boolean locally separated toposes and monoidal monotone complete C^*-categories. *arXiv preprint arXiv :1501.07045*, 2015.

[16] Simon Henry. Algebraic models of homotopy types and the homotopy hypothesis. *arXiv preprint arXiv :1609.04622*, 2016.

[17] Simon Henry. Localic metric spaces and the localic gelfand duality. *Advances in Mathematics*, 294 :634 – 688, 2016.

[18] Simon Henry. Measure theory over boolean toposes. *Mathematical Proceedings of the Cambridge Philosophical Society*, 2016.

[19] Simon Henry. The convolution algebra of an absolutely locally compact topos. *arXiv preprint arXiv :1701.00113*, 2017.

[20] Simon Henry. The localic istropy group of a topos. *arXiv preprint arXiv :1706.04835*, 2017.

[21] Simon Henry. Non-unital polygraphs form a presheaf category. *arXiv preprint arXiv :1711.00744*, 2017.

[22] Chris Heunen, Klaas Landsman, and Bas Spitters. The principle of general tovariance. 2008.

[23] Peter T. Johnstone. The point of pointless topology. *Bulletin (New Series) of the American Mathematical Society*, 8(1) :41–53, 1983.

[24] Mikhail M Kapranov and Vladimir A. Voevodsky. ∞-groupoids and homotopy types. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, 32(1) :29–46, 1991.

[25] Nicolaas Pieter Landsman. Lie groupoid C^*-algebras and Weyl quantization. *Communications in mathematical physics*, 206(2) :367–381, 1999.

[26] Emily Riehl and Michael Shulman. A type theory for synthetic ∞-categories. *arXiv preprint arXiv :1705.07442*, 2017.

[27] Hsian-Hua Tseng and Chenchang Zhu. Integrating lie algebroids via stacks. *Compositio Mathematica*, 142(1) :251, 2006.

[28] Benno Van Den Berg and Richard Garner. Types are weak ω-groupoids. *Proceedings of the London Mathematical Society*, 102(2) :370–394, 2011.

[29] Vladimir Voevodsky et al. Homotopy type theory : Univalent foundations of mathematics. *Institute for Advanced Study (Princeton), The Univalent Foundations Program*, 2013.